ACCELERATOR PHYSICS

Melbourne

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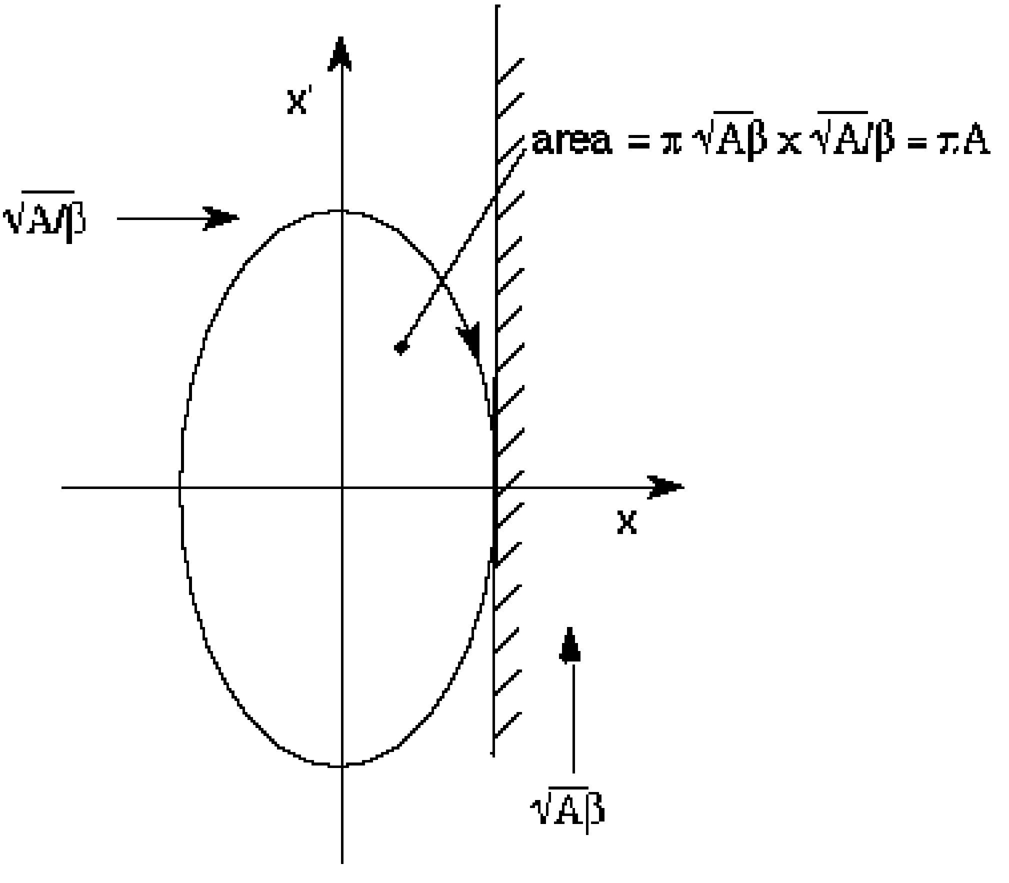
Previous lecture – Magnets Summary

- **Magnet types**
- **Multipole field expansion**
- **Taylor series expansion**
- **Dipole bending magnet**
- **Diamond quadrupole**
- **Various coil and yoke designs**
- **Power consumption of a magnet**
- **Magnet cost v. field**
- **Coil design geometry**
- **Field quality**
- **Shims extend the good field**
- **Flux density in the yoke**
- **Magnet ends**
- **Superconducting magnets**
- **Magnetic rigidity**
- **Bending Magnet**
- **Fields and force in a quadrupole**

Lecture 5 - Imperfections - Contents

- **Acceptance**
- **Making an orbit bump grow**
- **Circle diagram**
- **Closed orbit in the circle diagram**
- **Uncorrelated errors**
- **Sources of distortion**
- **FNAL measurement**
- **Diad bump**
- **Overlapping beam bumps**
- **Effect of quadrupole errors.**
- **Chromaticity**
- **Closed orbit in the circle diagram**
- **Gradient errors**
- **Working daigram**

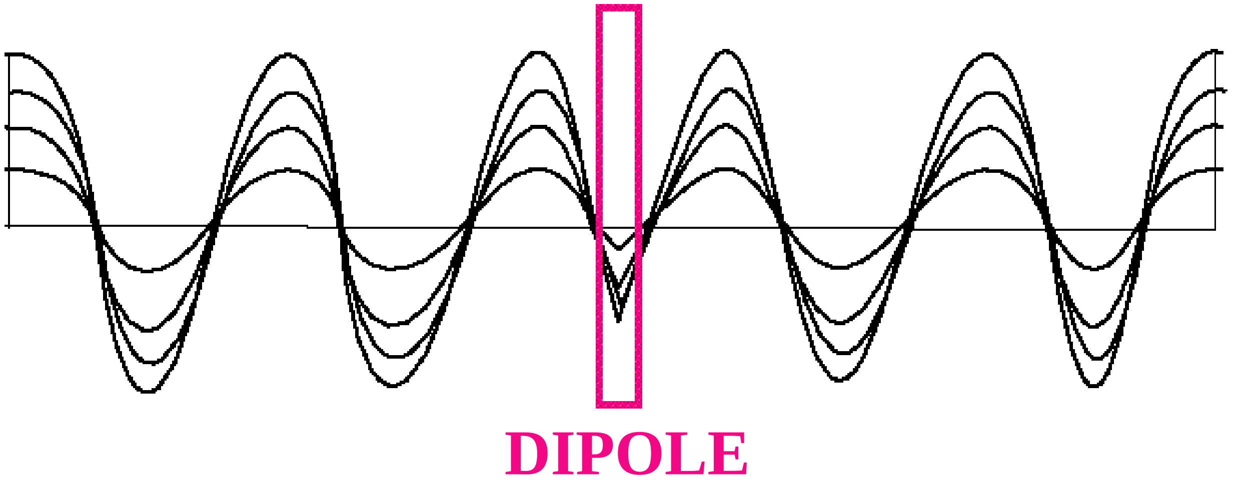
Acceptance



- **Largest particle grazing an obstacle defines acceptance.**
- **Acceptance is equivalent to emittance**

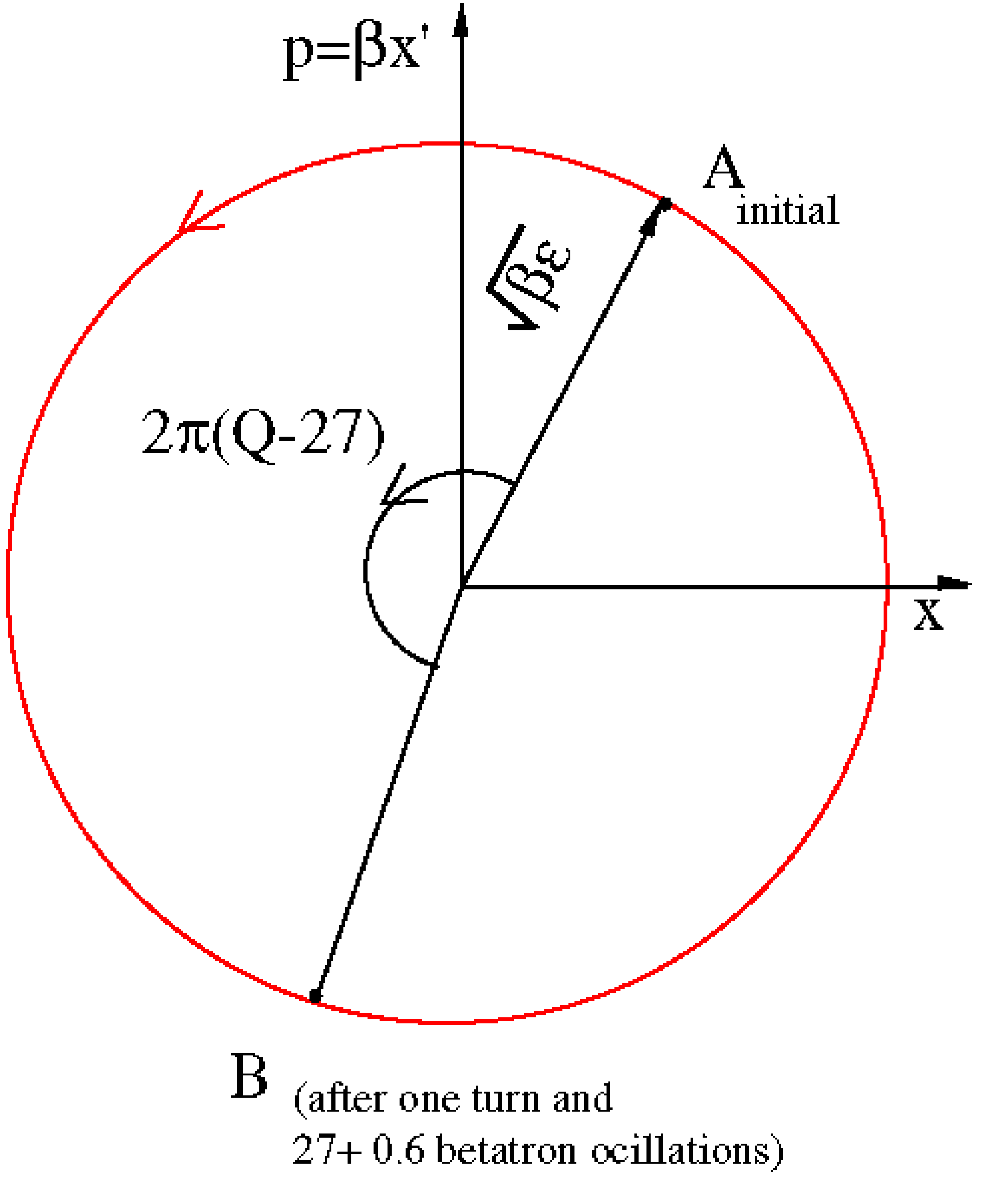
$$A = \frac{\hat{x}^2}{\beta}$$

Making an orbit bump grow



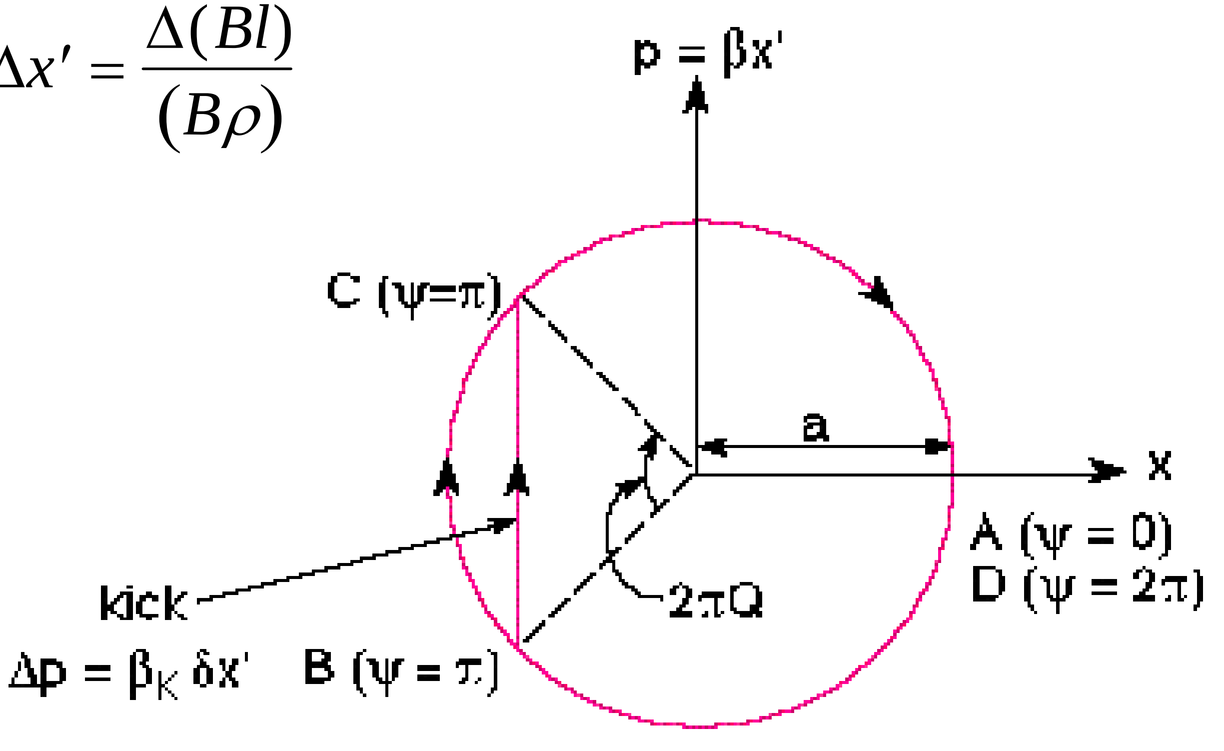
- As we slowly raise the current in a dipole:
- The zero-amplitude betatron particle follows a distorted orbit
- The distorted orbit is CLOSED
- It is still obeying Hill's Equation
- Except at the kink (dipole) it follows a betatron oscillation.
- Other particles with finite amplitudes oscillate about this new closed orbit

Circle diagram



Closed orbit in the circle diagram

$$\Delta x' = \frac{\Delta(Bl)}{(B\rho)}$$



Tracing a closed orbit for one turn in the circle diagram with a single kick. The path is ABCD.

$$\frac{\Delta p}{2} = \frac{\beta_k \delta x'}{2} = a \sin\left(\frac{2\pi Q}{2}\right)$$

$$a = \frac{\beta_k \delta x'}{2\sin \pi Q}$$ **elsewhere** $$\hat{x} = a\sqrt{\frac{\beta(s)}{\beta_k}} = \frac{\sqrt{\beta_k \beta(s)}}{2\sin \pi Q}\delta x'$$

Uncorrelated errors

- **A random distribution of dipole errors**

- **Take the r.m.s. average of** $\delta y_i' = \Delta(Bl)/(B\rho)$
- **Weighted according to the** β_k **values**
- **The expectation value of the amplitude is:**

$$\langle y(s) \rangle = \frac{\sqrt{\beta(s)}}{2\sqrt{2}\sin \pi Q} \sqrt{\sum_i \beta_i \delta y_i'^2}$$

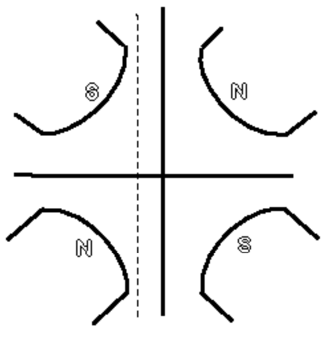
- **Kicks from the *N* magnets in the ring.**

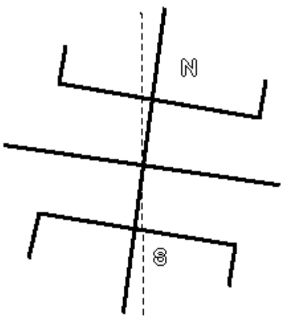
$$\approx \frac{\sqrt{\beta(s)\overline{\beta}}}{2\sqrt{2}\sin \pi Q} \sqrt{N} \frac{(\Delta B\ell)_{rms}}{B\rho}$$

- **The factor** $\sqrt{2}$ **takes into account the averaging over both sine and cosine phases**

- **A further factor 2 safety is applied to include 98% of all sample distributions.**

Sources of distortion





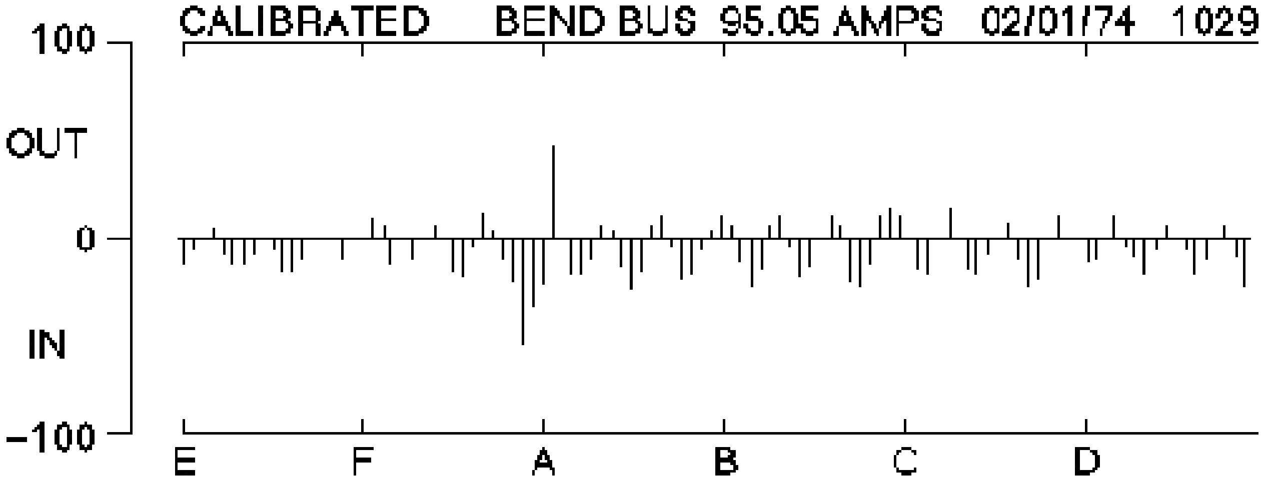
Δy **Δ**

Table 1

Sources of Closed Orbit Distortion

Type of element	Source of kick	r.m.s. value	$\langle \Delta B l / (B\rho) \rangle_{rms}$	plane
Gradient magnet	Displacement	$<\Delta y>$	$k_i l_i <\Delta y>$	x,z
Bending magnet (bending angle = θi)	Tilt	$<\Delta>$	$\theta_i <\Delta>$	z
Bending magnet	Field error	$<\Delta B/B>$	$\theta i <\Delta B/B>$	x
Straight sections (length = d_i)	Stray field	$<\Delta B_S>$	$d_i \langle \Delta B_s \rangle / (B\rho)_{inj}$	x,z

FNAL MEASUREMENT



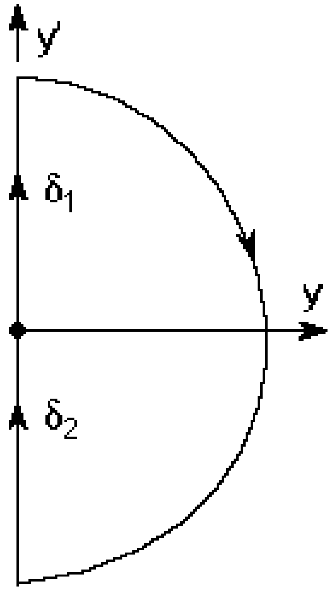
- Historic measurement from FNAL main ring
- Each bar is the position at a quadrupole
- +/- 100 is width of vacuum chamber
- Note mixture of 19th and 20th harmonic
- The Q value was 19.25

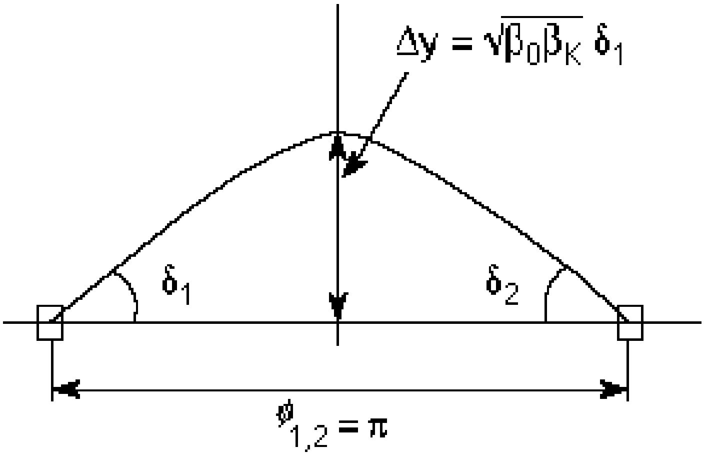
Diad bump

- **Simplest bump is from two equal dipoles 180 degrees apart in betatron phase. Each gives:**

$$\delta = \frac{\Delta(B\,l)}{B\rho}$$

- **The trajectory is :** $y(s) = \delta\sqrt{\beta(s)\beta_k}\sin(\phi - \phi_0)$

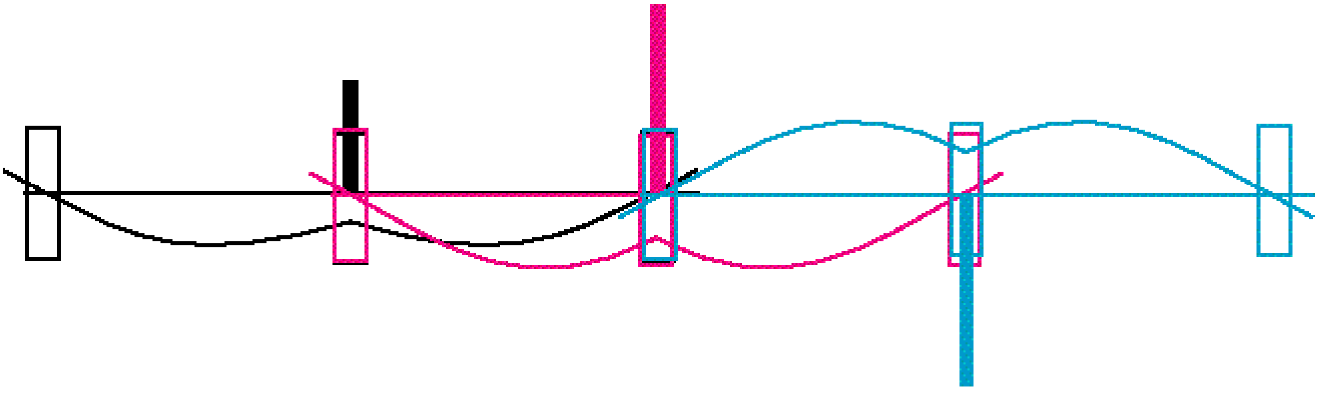




- **The matrix is**

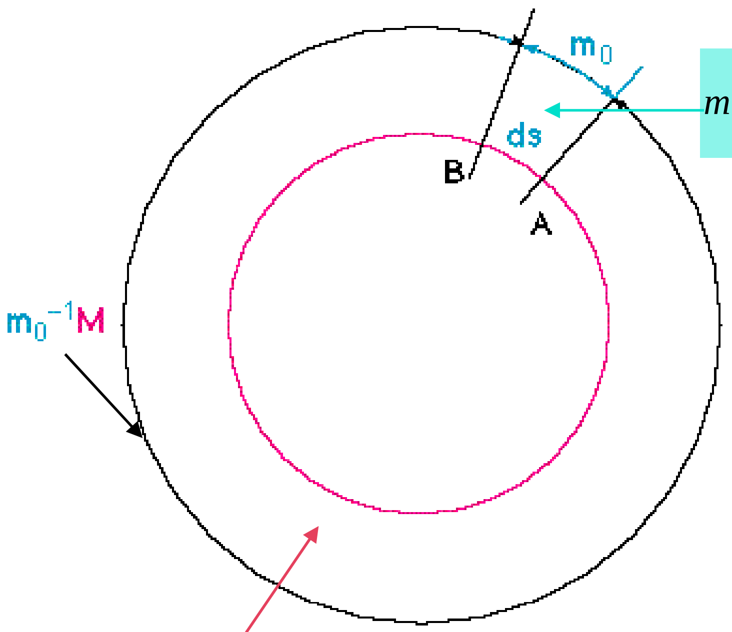
$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \left(\sqrt{\beta}/\sqrt{\beta_0}\right)\left(\cos\Delta\phi + \alpha_0 \sin\Delta\phi\right) & , \sqrt{\beta_0\beta}\sin\Delta\phi \\ \left(-1/\sqrt{\beta_0\beta}\right)\left\{(\alpha - \alpha_0)\cos\Delta\phi + (1 + \alpha\alpha_0)\sin\Delta\phi\right\}, & \left(\sqrt{\beta}/\sqrt{\beta_0}\right)\left(\cos\Delta\phi - \alpha\sin\Delta\phi\right) \end{pmatrix} \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$

Overlapping beam bumps



- Each colour shows a triad bump centred on a beam position measurement.
- A computer calculates the superposition of the currents in the dipoles and corrects the whole orbit simultaneously

Gradient errors



$$m_0 = \begin{pmatrix} 1 & 0 \\ -k_0(s)ds & 1 \end{pmatrix} ,$$

$$m = \begin{pmatrix} 1 & 0 \\ -\left[k_0(s) + \delta k(s)\right]ds & 1 \end{pmatrix} .$$

$$M_0(s) = \begin{pmatrix} \cos\varphi_0 + \alpha_0 \sin\varphi_0 , & \beta_0 \sin\varphi_0 \\ -\gamma_0 \sin\varphi_0 , & \cos\varphi_0 - \alpha_0 \sin\varphi_0 \end{pmatrix} .$$

$$M(s) = \boxed{m m_0^{-1}} M_0 .$$

$$m m_0^{-1} = \begin{pmatrix} 1 & 0 \\ -\delta k(s_1)ds & 1 \end{pmatrix} .$$

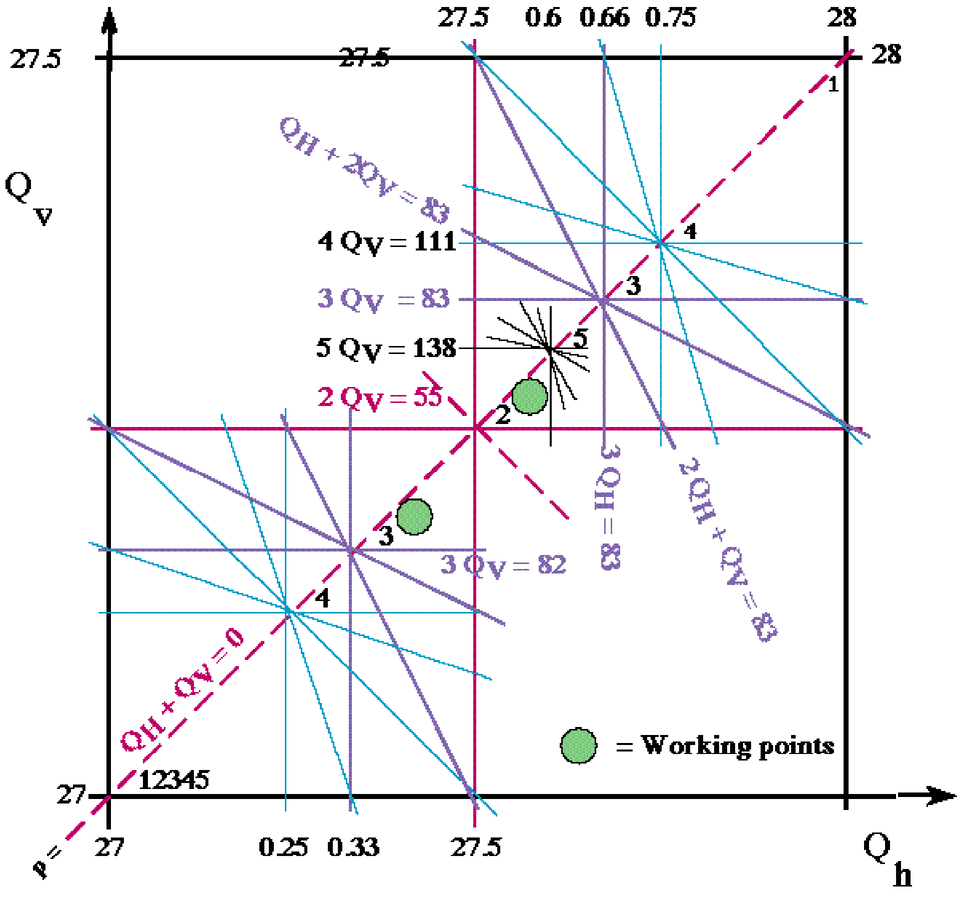
$$M = \begin{pmatrix} \cos\phi_0 + \alpha_0 \sin\phi_0 , & \beta_0 \sin\phi_0 \\ -\delta k(s)ds\left(\cos\phi_0 + \alpha_0 \sin\phi_0\right) - \gamma \sin\phi_0 , & -\delta k(s)ds\beta_0 \sin\phi_0 + \cos\phi_0 - \alpha_0 \sin\phi_0 \end{pmatrix} .$$

$$\Delta(\mathrm{Tr}\ \mathrm{M})/2 = \Delta(\cos\phi) = -\Delta\phi \sin\phi_0 = \frac{\sin\phi_0}{2}\beta_0(s)\delta k(s)ds$$

$$2\pi\Delta Q = \Delta\phi = \frac{\beta(s)\delta k(s)ds}{2} .$$

$$\boxed{\Delta Q = \frac{1}{4\pi}\int \beta(s)\delta k(s)\ ds .}$$

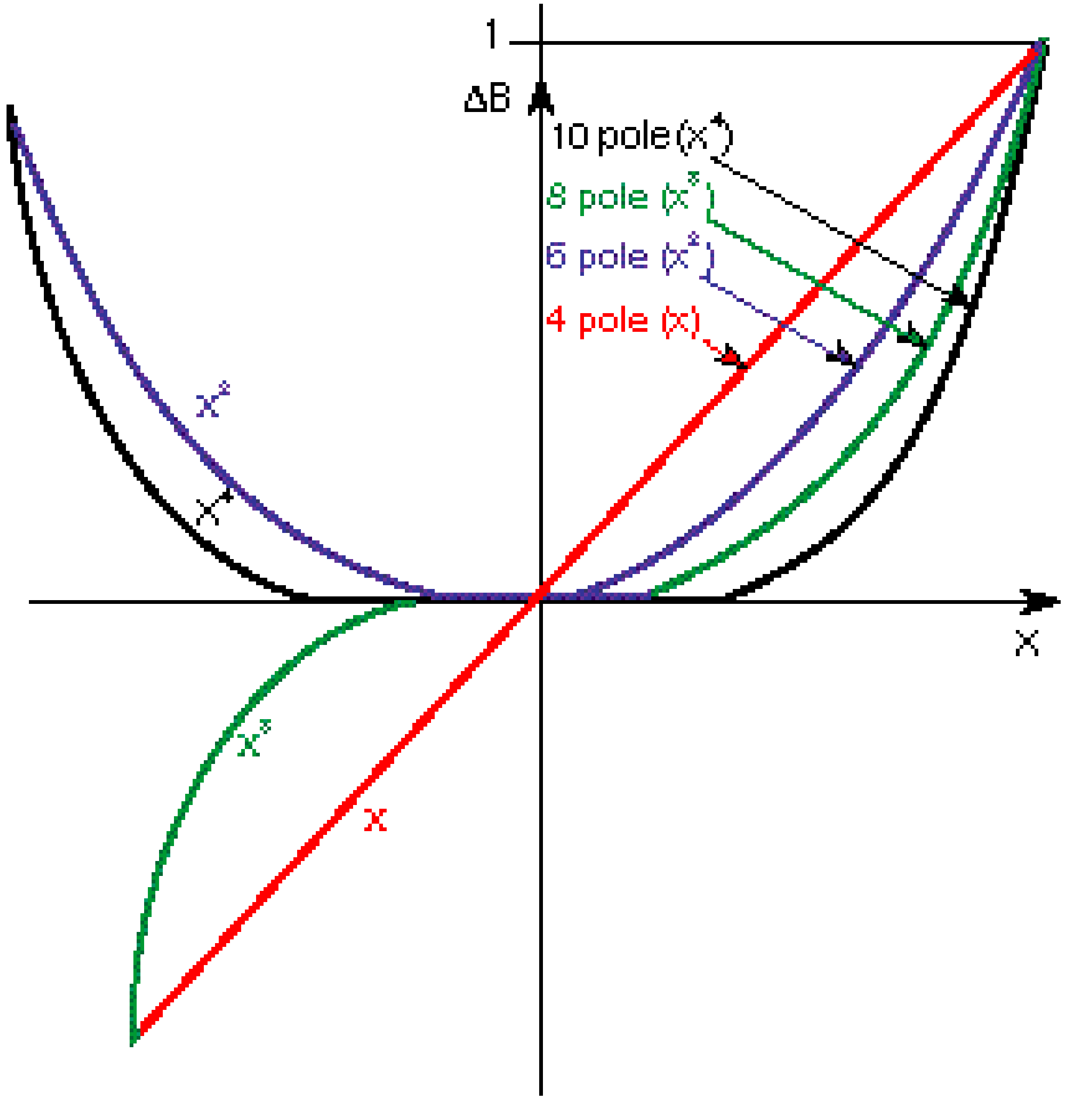
Q diagram



$$nQ = p \ ,$$

$$1Q_H + mQ_V = p \ ,$$

Multipole field shapes



Physics of Chromaticity

- **The Q is determined by the lattice quadrupoles whose strength is:**

$$k = \frac{1}{(B\rho)}\frac{dB_z}{dx} \propto \frac{1}{p}$$

- **Differentiating:** $$\frac{\Delta k}{k} = -\frac{\Delta p}{p} \; .$$
- **Remember from gradient error analysis**

$$\Delta Q = \frac{1}{4\pi}\int \beta(s)\delta k(s)\ ds \; .$$

- **Giving by substitution**

$$\Delta Q = \frac{1}{4\pi}\int \beta(s)\Delta k(s)ds = \left[\frac{-1}{4\pi}\int \beta(s)k(s)ds\right]\frac{\Delta p}{p} \; .$$

$$\Delta Q = Q'\frac{\Delta p}{p}$$

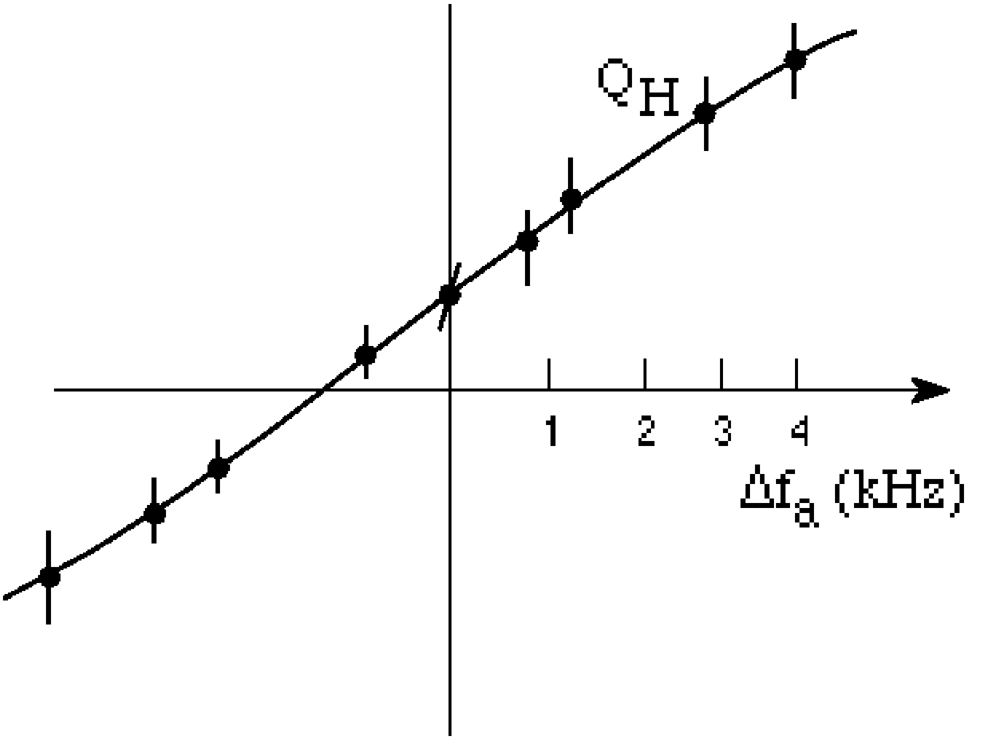
***Q'* is the chromaticity**

- **"Natural" chromaticity**

$$Q' = -\frac{1}{4\pi}\oint \beta(s)k(s)ds \approx -1.3Q$$

N.B. Old books say $\xi = \frac{p}{Q}\frac{dQ}{dp} = \frac{Q'}{Q}$

Measurement of Chromaticity



- **We can steer the beam to a different mean radius and a different momntum by changing the rf frequency and measure Q**

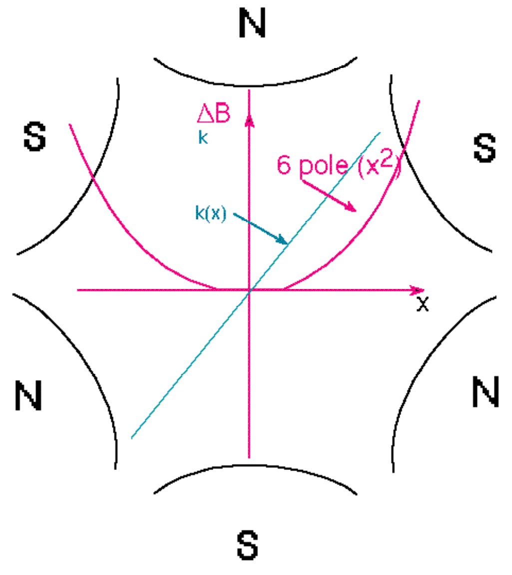
$$\Delta f_a = f_a \eta \frac{\Delta p}{p} \qquad \Delta r = D_{av} \frac{\Delta p}{p}$$

- **Since** $\Delta Q = Q' \frac{\Delta p}{p}$

- **Hence**

$$\therefore Q' = f_a \eta \frac{dQ}{df_a}$$

Correction of Chromaticity



- **Parabolic field of a 6 pole is really a gradient which rises linearly with x**
- **If x is the product of momentum error and dispersion**

$$\Delta k = \frac{B''D}{(B\rho)}\frac{\Delta p}{p} .$$

- **The effect of all this extra focusing cancels chromaticity**

$$\Delta Q = \left[\frac{1}{4\pi}\int\frac{B''(s)\beta(s)D(s)ds}{(B\rho)}\right]\frac{dp}{p} .$$

- **Because gradient is opposite in v plane we must have two sets of opposite polarity at F and D quads where betas are different**

Lecture 7 - Beams and Errors - Summary

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