Lecture 6

ACCELERATOR PHYSICS

Melbourne

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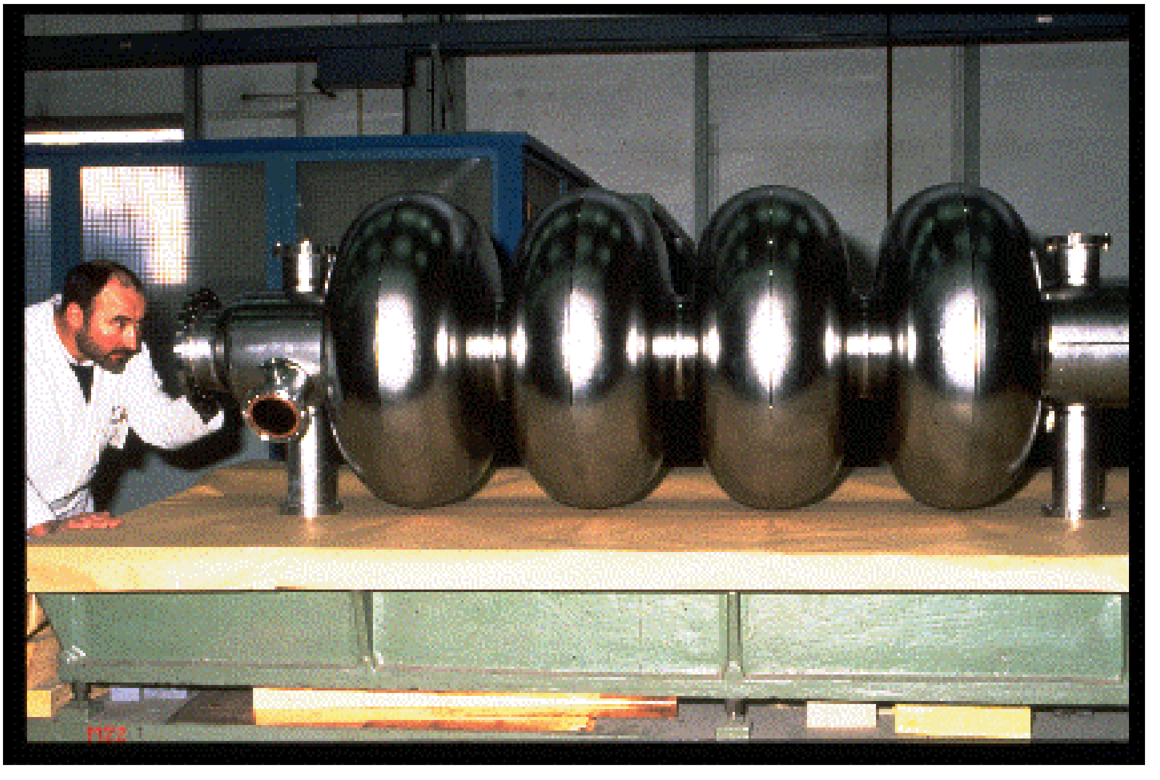
Recap of previous lecture - Imperfections

- Acceptance
- Making an orbit bump grow
- Circle diagram
- Closed orbit in the circle diagram
- Uncorrelated errors
- Sources of distortion
- FNAL measurement
- Diad bump
- Overlapping beam bumps
- Effect of quadrupole errors.
- Chromaticity
- Closed orbit in the circle diagram
- Gradient errors
- Working daigram

Lecture 6 - Longitudinal dynamics I - contents

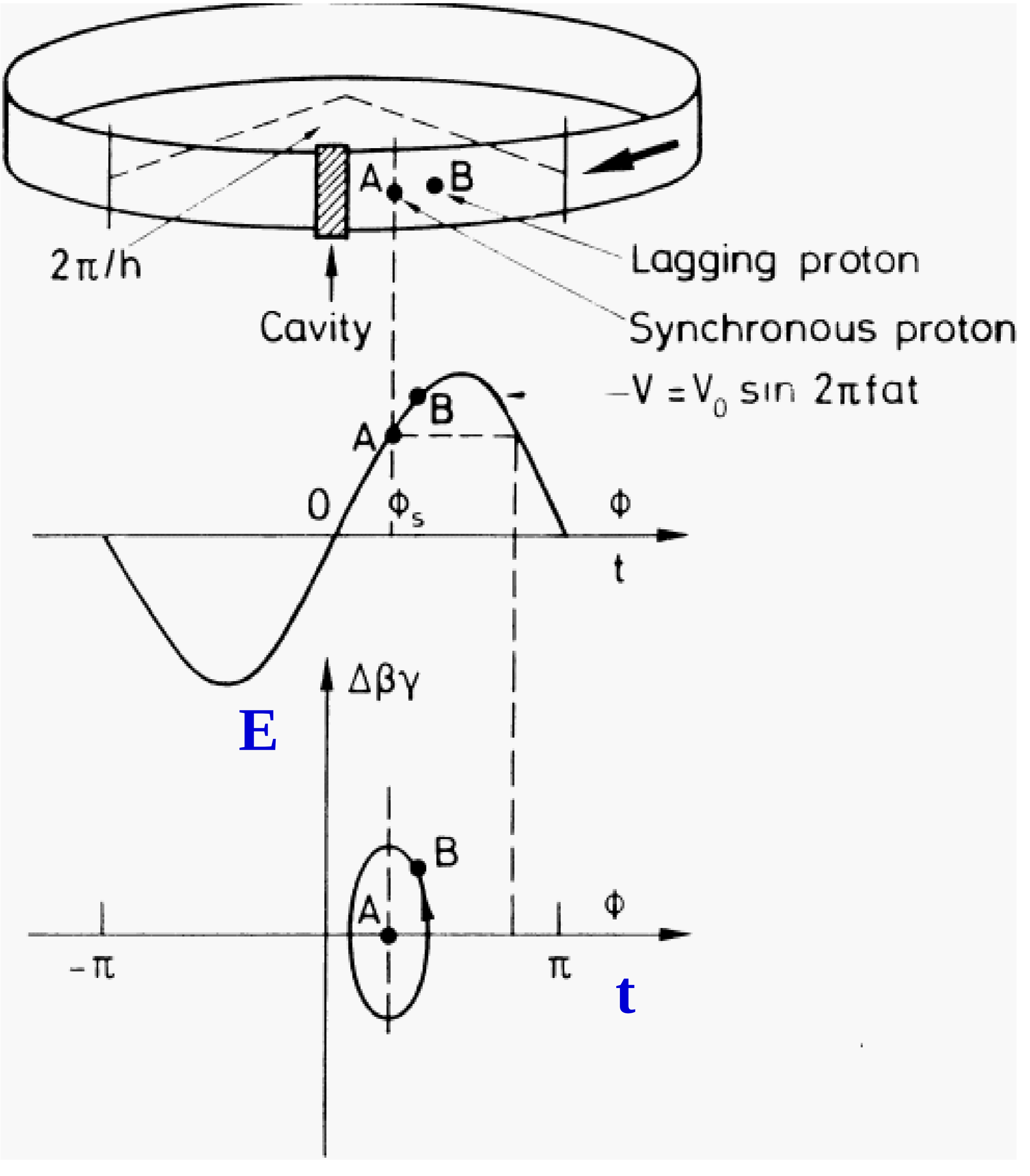
- RF Cavity Cells
- Phase stability
- Bucket and pendulum
- Closed orbit of an ideal machine
- Analogy with gravity
- Dispersion
- Dispersion in the SPS
- Dispersed beam cross sections
- Dispersion in a bend (approx)
- Dispersion – from the “sine and cosine” trajectories
- From “three by three” matrices

RF Cavity Cells

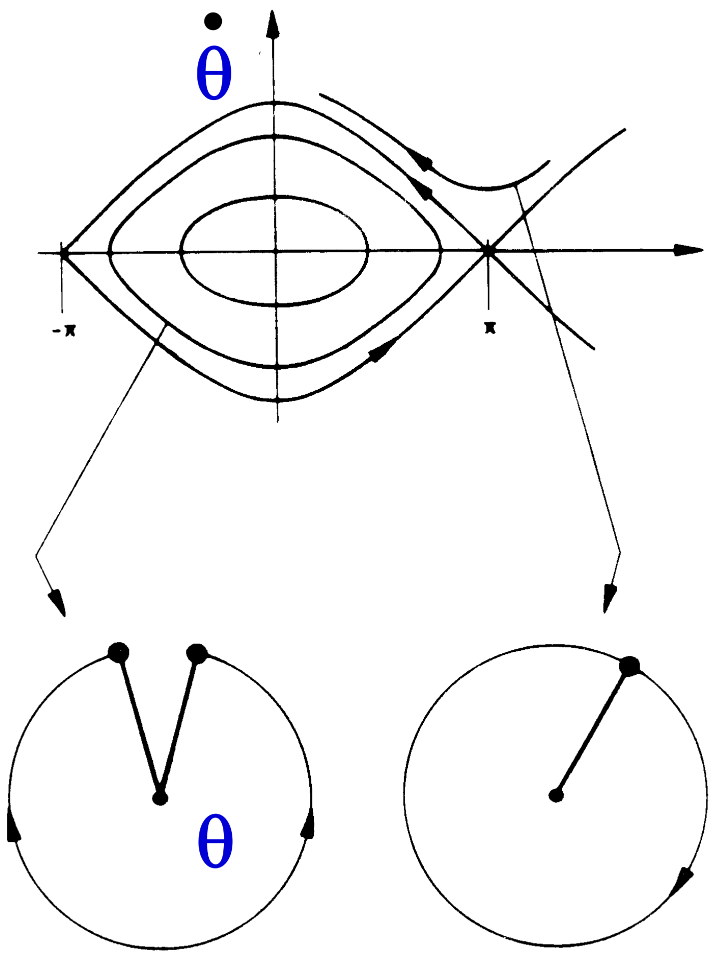


CAV.GIF

Phase stability

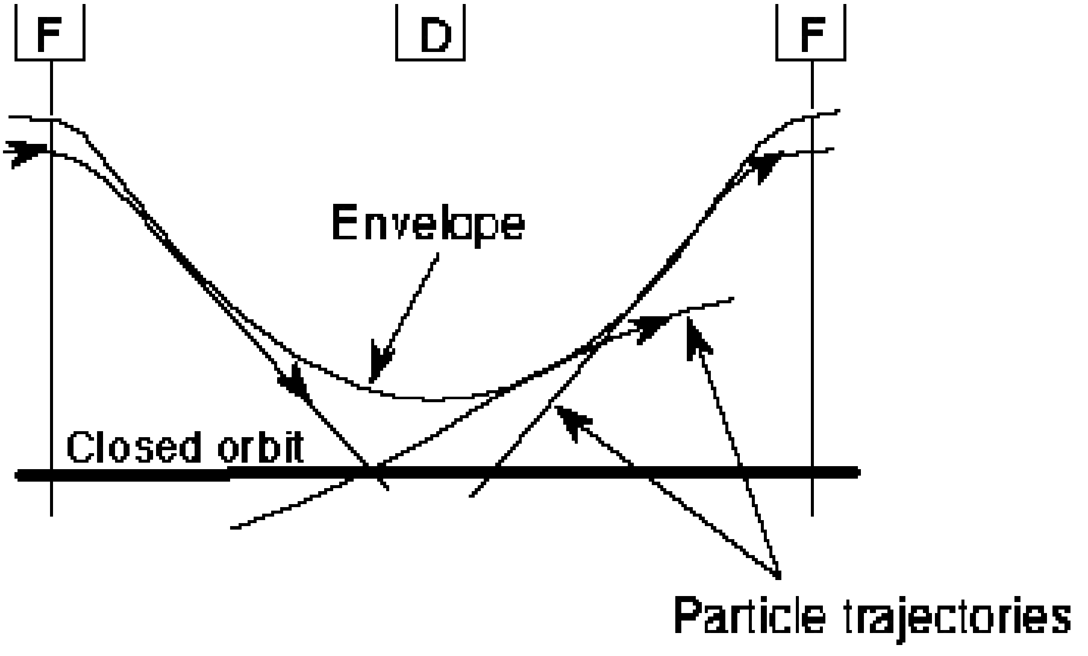


Bucket and pendulum

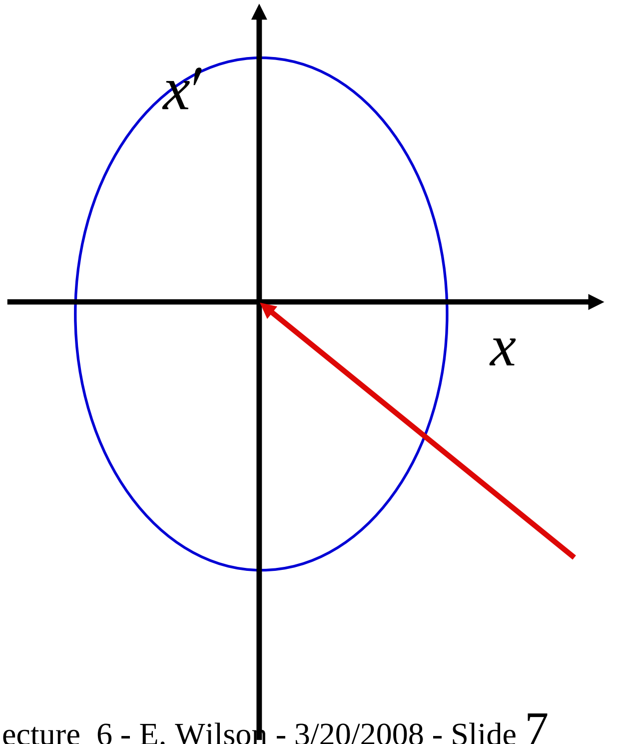


- The “bucket” of synchrotron motion is just that of the rigid pendulum
- Linear motion at small amplitude
- Metastable fixed point at the top
- Continuous rotation outside

Closed orbit of an ideal machine

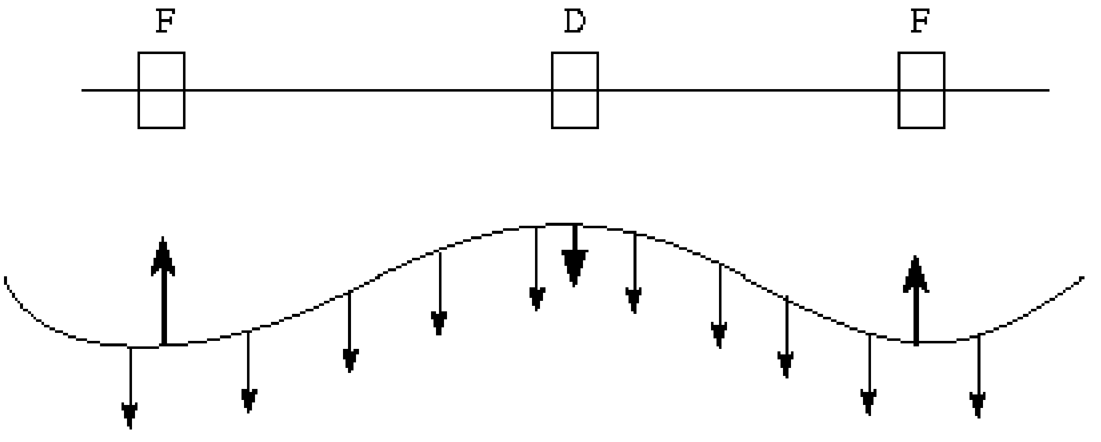


- In general particles executing betatron oscillations have a finite amplitude
- One particle will have zero amplitude and follows an orbit which closes on itself
- In an ideal machine this passes down the axis



Closed orbit
Zero betatron amplitude

Analogy with gravity



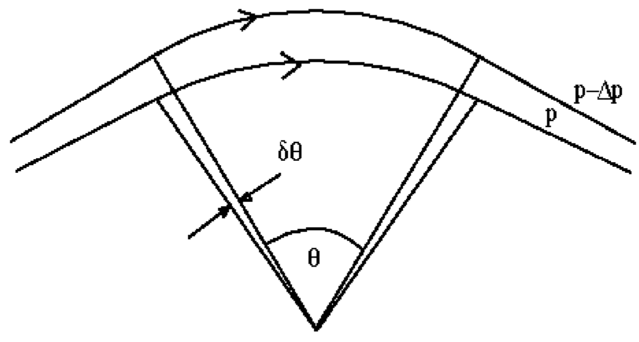
- What keeps particles in the machine
- There is a solution to Hills Equation
- It is closed and symmetric
- It is closer to the axis at vertically Defocusing Quadrupoles

$$\Delta z' = \frac{\Delta\ell\,\beta}{(\beta\rho)} = k\ell z$$

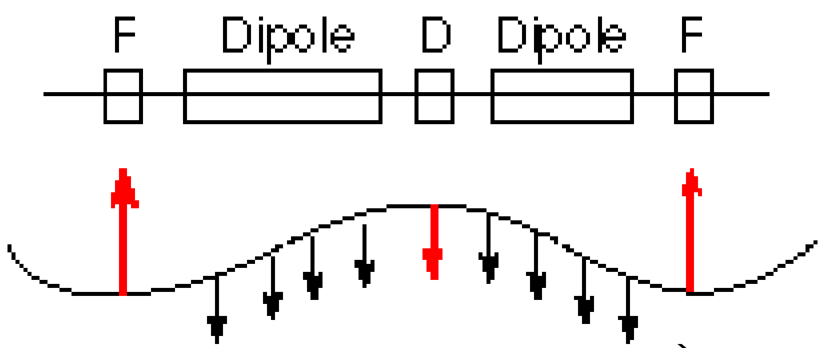
- Deflection is larger in F than D and cancels the force of gravity elsewhere.
- We could call the shape the "suspension" function.

Fig. cas 1.4C

Dispersion



- Low momentum particle is bent more
- It should spiral inwards but:
- There is a displaced (inwards) closed orbit
- Closer to axis in the D's
- Extra (outward) force balances extra bends

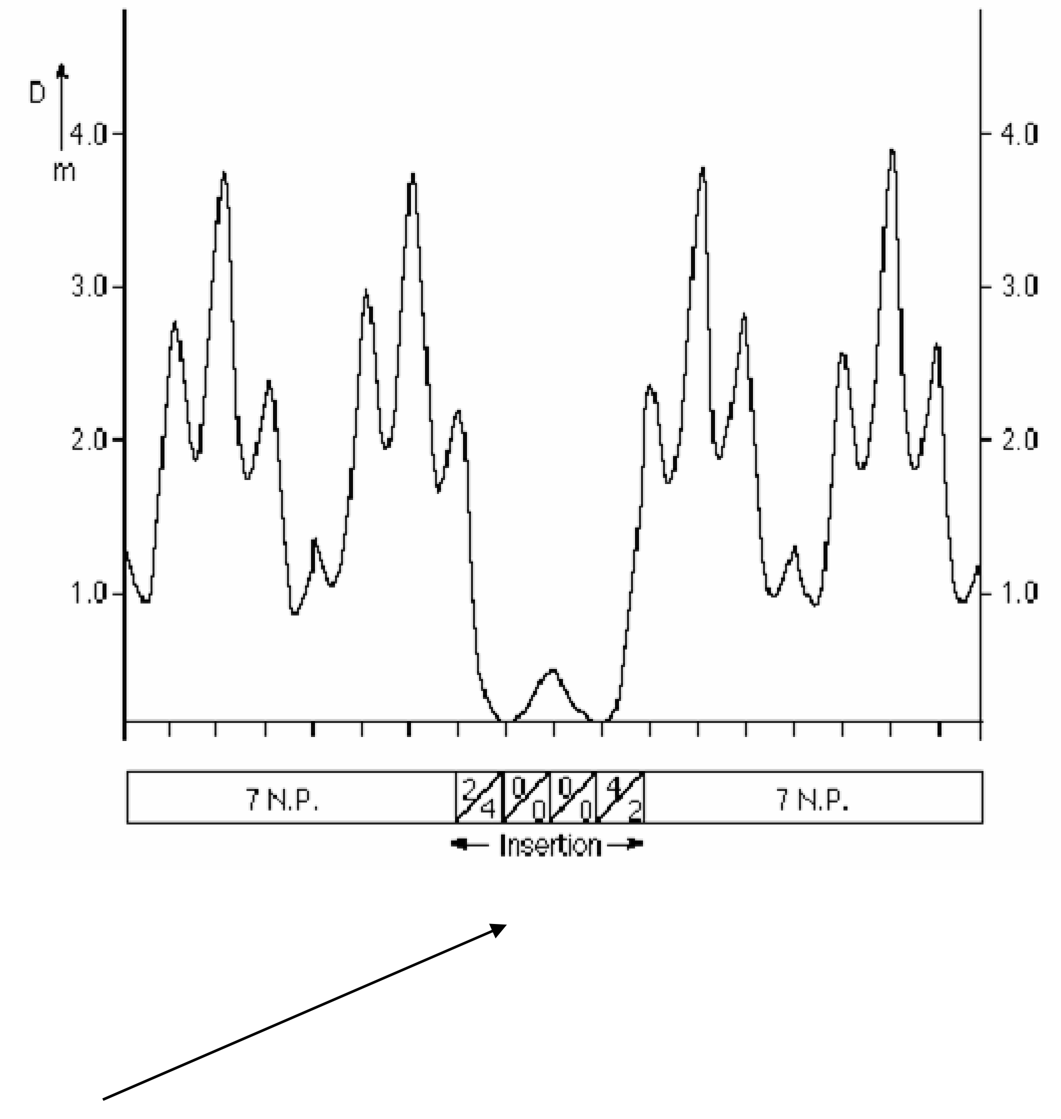


- *D(s)* is the "dispersion function"

$$x = D(s)\frac{\Delta p}{p}$$

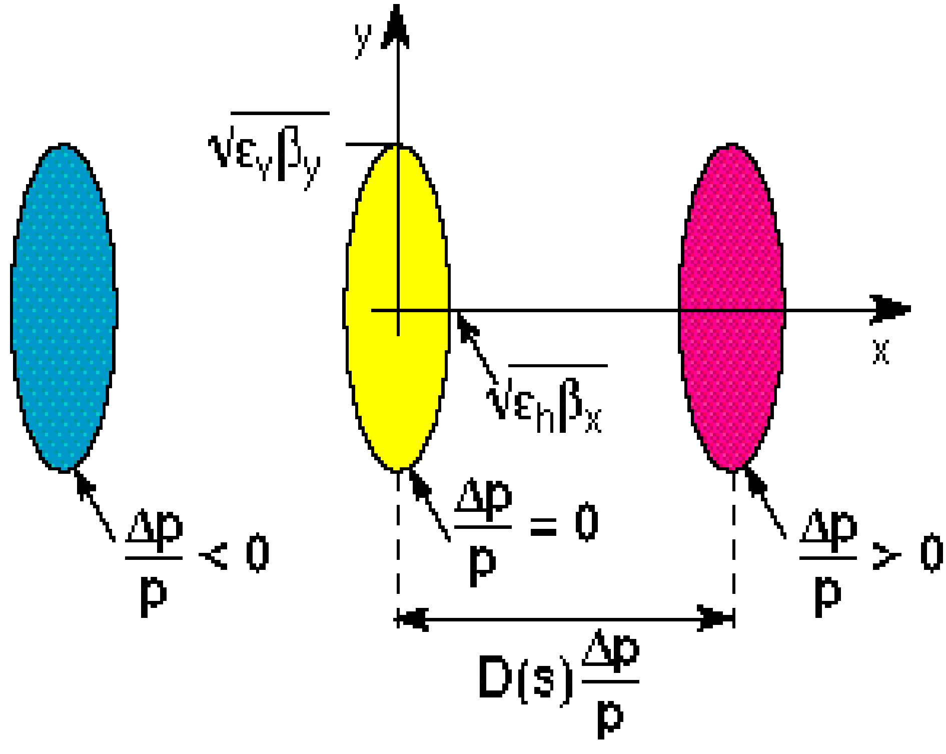
Fig. cas 1.7-7.1C

Dispersion in the SPS



- This is the long straight section where dipoles are omitted to leave room for other equipment - RF - Injection - Extraction, etc
- The pattern of missing dipoles in this region indicated by “0” is chosen to control the Fourier harmonics and make *D(s)* small
- It doesn’t matter that it is big elsewhere

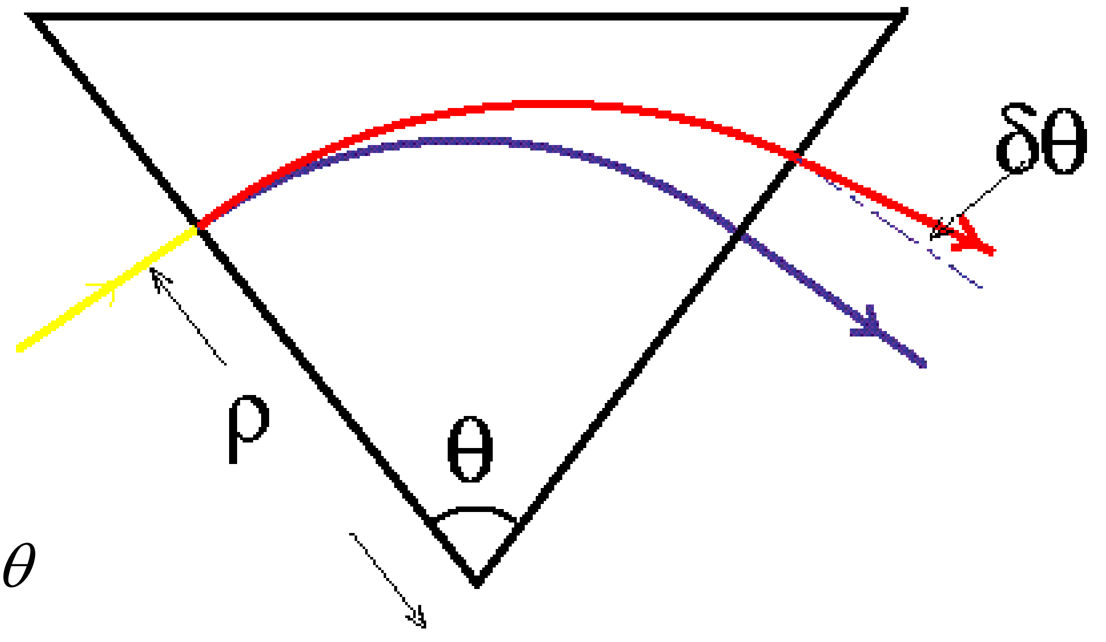
Dispersed beam cross sections



- **These are real cross-section of beam**
- **The central and extreme momenta are shown**
- **There is of course a continuum between**
- **The vacuum chamber width must accommodate the full spread**
- **Half height and half width are:**

$$a_V = \sqrt{\beta_V \varepsilon_V} \ , \quad a_H = \sqrt{\beta_H \varepsilon_H} + D(s)\frac{\Delta p}{p} \ .$$

Dispersion in a bend (approx)



$$\delta\theta = -\frac{dp}{p}\theta$$

$$x = \int_0^\theta x' ds = \rho\int_0^\theta x' d\theta = \left(\frac{dp}{p}\right)\rho\int_0^\theta \theta d\theta = \left(\frac{dp}{p}\right)\rho\left[\frac{\theta^2}{2}\right]_0^\theta$$

$$\begin{pmatrix} x_2 \\ x_2' \\ \frac{\delta p}{p} \end{pmatrix} = \begin{pmatrix} 1 & \rho\theta & \frac{\rho\theta^2}{2} \\ \frac{-\theta}{\rho} & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \\ \frac{\delta p}{p} \end{pmatrix}$$

Bender.adb

Dispersion – from the "sine and cosine" trajectories

- **The combination of diplacement, divergence and dispersion gives:**

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0} + \frac{\Delta p}{p} \begin{pmatrix} D \\ D' \end{pmatrix}$$

- **Expressed as a matrix**

$$\begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_{s_0}$$

- **It can be shown that:**

$$\boxed{D(s) = S(s)\int_{s_0}^{s} \frac{1}{\rho(t)} C(t)\,dt - C(s)\int_{s_0}^{s} \frac{1}{\rho(t)} S(t)\,dt}$$

- **Fulfils the particular solution of Hill's eqn. when forced :**

$$D''(s) + K(s)D(s) = \frac{1}{\rho(s)}$$

From “three by three” matrices

- **Adding momentum defect to horizontal divergence and displacement vector–**

$$\begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_2 = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \Delta p/p \end{pmatrix}_1$$

- **Compute the ring as a product of small matrices and then use:**

$$D'(s) = \frac{m_{13}m_{21} + (1 - m_{11})m_{23}}{(1 - m_{11})(1 - m_{22}) - m_{21}m_{12}}$$

$$D(s) = \left(\frac{m_{13} - m_{12}D'(s)}{1 - m_{11}} \right) D'(s)$$

- **To find the dispersion vector at the starting point**
- **Repeat for other points in the ring**

Lecture 7 - Transverse Dynamics – Summary

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