Lecture 7

ACCELERATOR PHYSICS

Melbourne

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Recap of previous lecture - Longitudinal dynamics I

- **RF Cavity Cells**
- **Phase stability**
- **Bucket and pendulum**
- **Closed orbit of an ideal machine**
- **Analogy with gravity**
- **Dispersion**
- **Dispersion in the SPS**
- **Dispersed beam cross sections**
- **Dispersion in a bend (approx)**
- **Dispersion – from the "sine and cosine" trajectories**
- **From "three by three" matrices**..

Lecture 7 - Longitudinal dynamics II -contents

- **Transition - does an accelerated particle catch up - it has further to go**
- **Phase jump at transition**
- **Synchrotron motion**
- **Synchrotron motion (continued)**
- **Large amplitudes**
- **Buckets**
- **Buckets**
- **Adiabatic capture**
- **A chain of buckets**

Transition - does an accelerated particle catch up - it has further to go

$$f = \frac{\beta c}{2\pi R}\,, \qquad (\beta = v/c)$$

Is a function of two, momentum dependent, terms β and R.

$$p = \frac{E_0\beta}{\sqrt{1-\beta^2}}\,. \quad \textbf{and} \quad R \approx R(\Delta p/p = 0) + D\frac{\Delta p}{p}$$

Using partial differentials to define a slip factor:

$$\frac{df}{dp} = \frac{\partial f}{\partial \beta}\frac{d\beta}{dp} + \frac{\partial f}{\partial R}\frac{dR}{dp}$$

$$\eta_{rf} = \frac{\Delta f / f}{\Delta p / p} = \frac{p}{\beta}\frac{d\beta}{dp} - \frac{p}{R}\frac{dR}{dp} = \frac{1}{\gamma^2} - \frac{\overline{D}}{R_0}$$

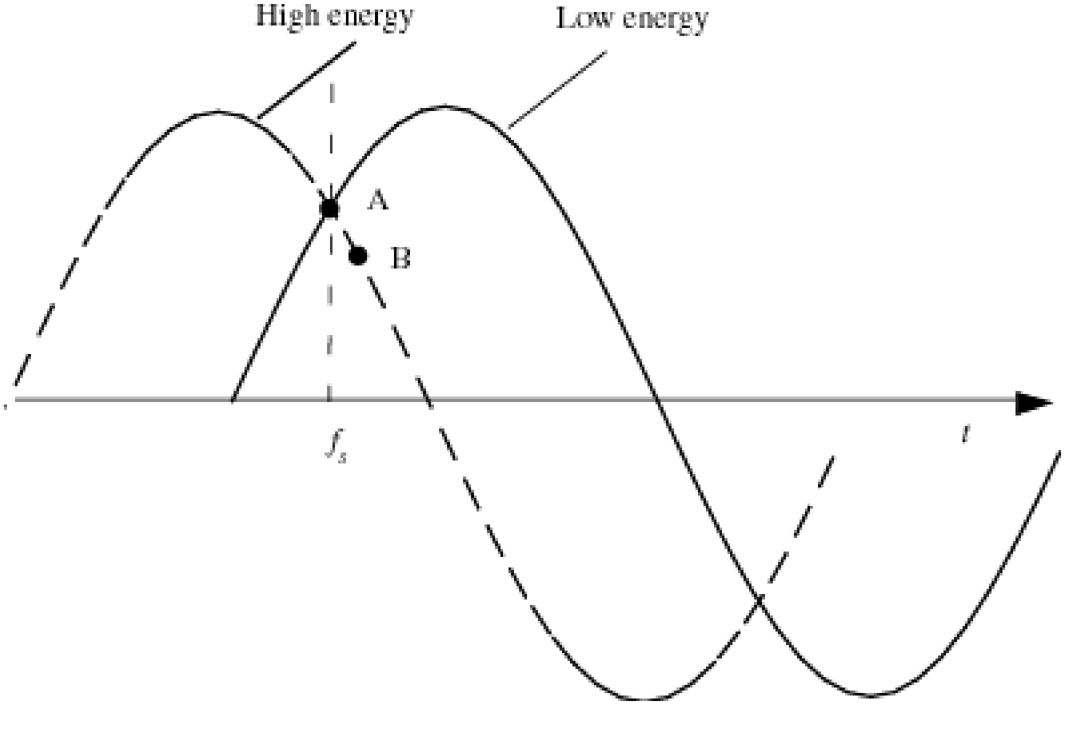
This changes from negative to positive and is zero at ‘ transition’ when:

GAMMA TRANSITION

$$\frac{1}{\gamma_{tr}^2} = \frac{\overline{D}}{R}\,.$$

Phase jump at transition

BECAUSE ..... $$\eta_{rf} == \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$$



Synchrotron motion

- **Recall** $p = m_0 c(\beta\gamma)$.
- **Elliptical trajectory for small amplitude**

$$\Delta(\beta\gamma) = \Delta(\beta\gamma)\cos 2\pi f_s t$$

$$\phi = \hat{\phi}\sin 2\pi f_s t$$

- **Note that frequency is rate of change of phase**

- **From definition of the slip factor η**

$$\dot{\phi} = 2\pi h\left[f(\beta\gamma) - f(0)\right] = 2\pi h \Delta f$$

- **Substitute and differentiate again**

$$\Delta f = \eta f \frac{\Delta p}{p} = \eta f \frac{\Delta(\beta\gamma)}{(\beta\gamma)} = \frac{\eta f}{\beta^2}\frac{\Delta\gamma}{\gamma} = \frac{\eta f}{E_0\beta^2\gamma}\Delta E$$

$$\ddot{\phi} = -\frac{2\pi h \eta f^2}{E_0\beta^2\gamma}(\Delta E)$$

- **But the extra acceleration is**

- **THUS** $\Delta E = V_0(\sin\phi - \sin\phi_s)$

$$\ddot{\phi} = -\frac{2\pi V_0 h \eta f^2}{E_0\beta^2\gamma}(\sin\phi - \sin\phi_s)$$

Synchrotron motion (continued)

- **This is a biased rigid pendulum**

$$\ddot{\phi} = -\frac{2\pi V_0 h \eta f^2}{E_0 \beta^2 \gamma}\left(\sin\phi - \sin\phi_s\right)$$

- **For small amplitudes**

$$\ddot{\phi} + \frac{2\pi V_0 h \eta f^2}{E_0 \beta^2 \gamma}\phi = 0$$

- **Synchrotron frequency**

$$f_s = \sqrt{\frac{|\eta| h V_0 \cos\phi_s}{2\pi E_0 \beta^2 \gamma}}\, f \;.$$

- **Synchrotron “tune”**

$$Q_s = \frac{f_s}{f} = \sqrt{\frac{|\eta| h V_0 \cos\phi_s}{2\pi E_0 \beta^2 \gamma}} \;.$$

Large amplitudes

$$\ddot{\phi} = -\frac{2\pi V_0 h \eta f^2}{E_0 \beta^2 \gamma}(\sin\phi - \sin\phi_s)$$

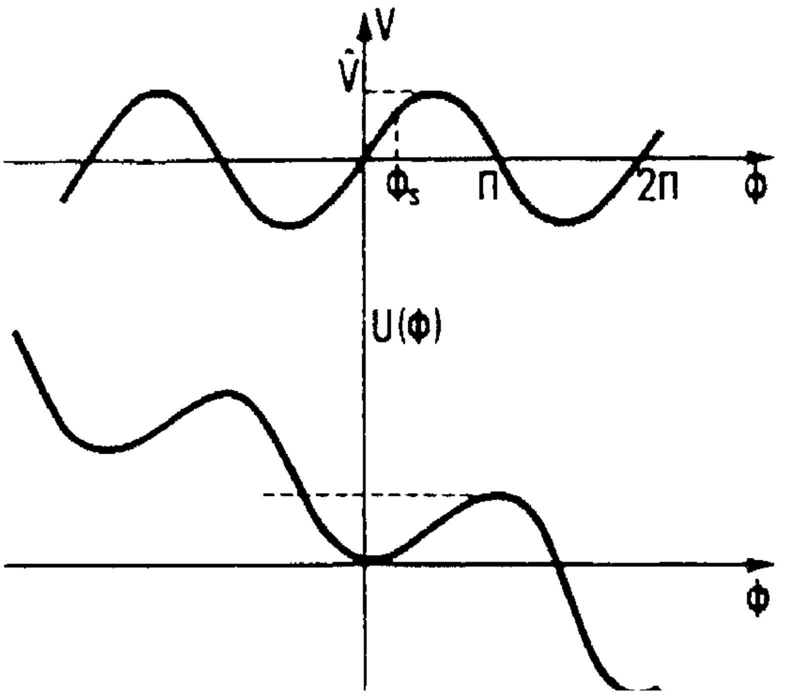
- **and** $\Omega_s = \sqrt{\frac{|\eta| h V_0 \cos\phi_s}{2\pi E_0 \beta^2 \gamma}}\,\omega_{rev}\,.$
- **become**

$$\ddot{\phi} = -\frac{\Omega_s^2}{\cos\phi_s}(\sin\phi - \sin\phi_s)$$

- **Integrated becomes an invariant**

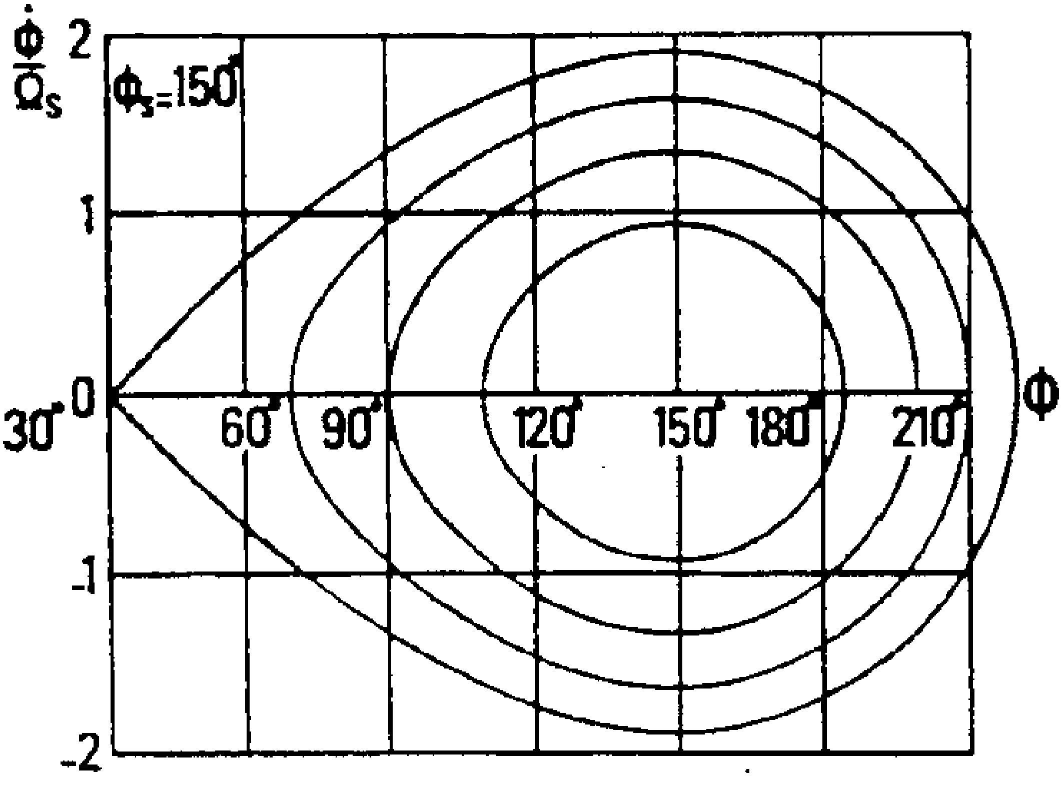
$$\frac{\dot{\phi}^2}{2} - \frac{\Omega_s^2}{\cos\phi_s}(\cos\phi + \phi\sin\phi_s) = const.$$

- **The second term is the potential energy function**



Buckets

- Seen from above this is a bucket (in phase space) for different values of ϕ_s

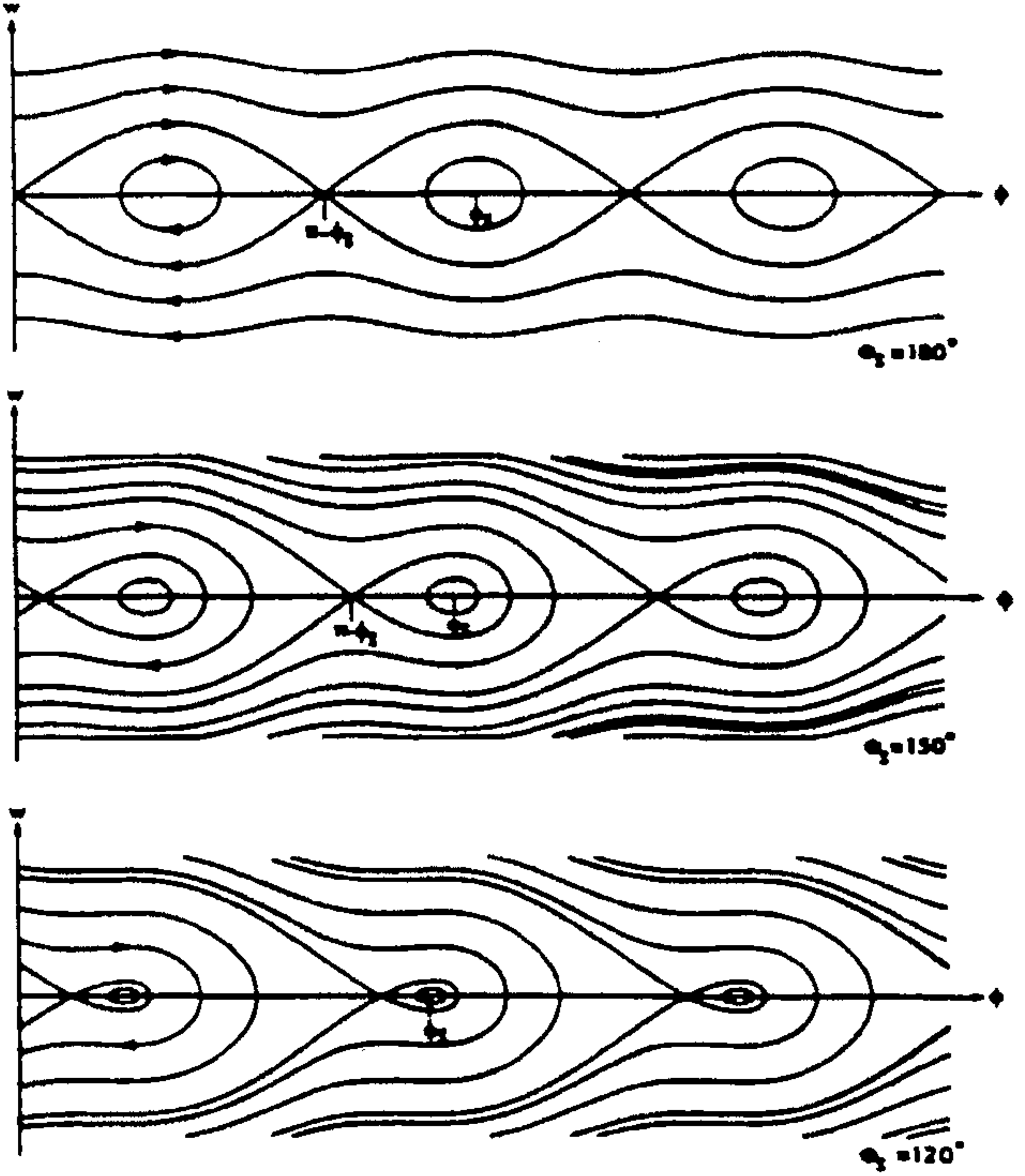


- The equation of each separatrix is

$$\frac{\dot{\phi}^2}{2} - \frac{\Omega_s^2}{\cos\phi_s}\left(\cos\phi + \phi\sin\phi_s\right) =$$

$$-\frac{\Omega_s^2}{\cos\phi_s}\left[\cos(\pi - \phi_s) + (\pi - \phi_s)\sin\phi_s\right].$$

A chain of buckets

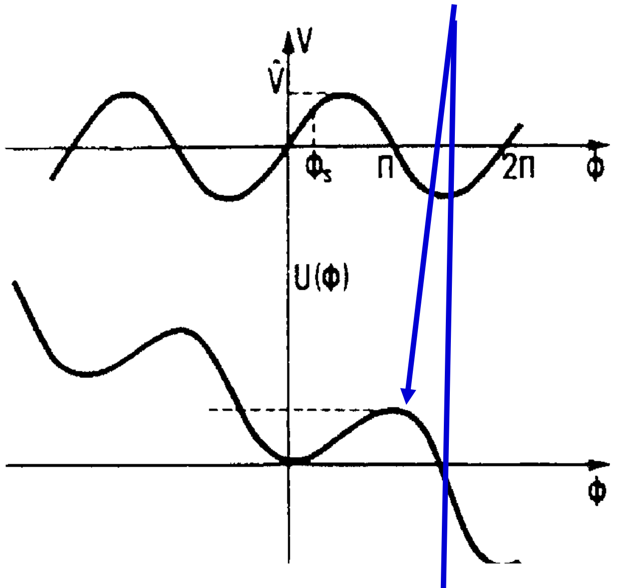


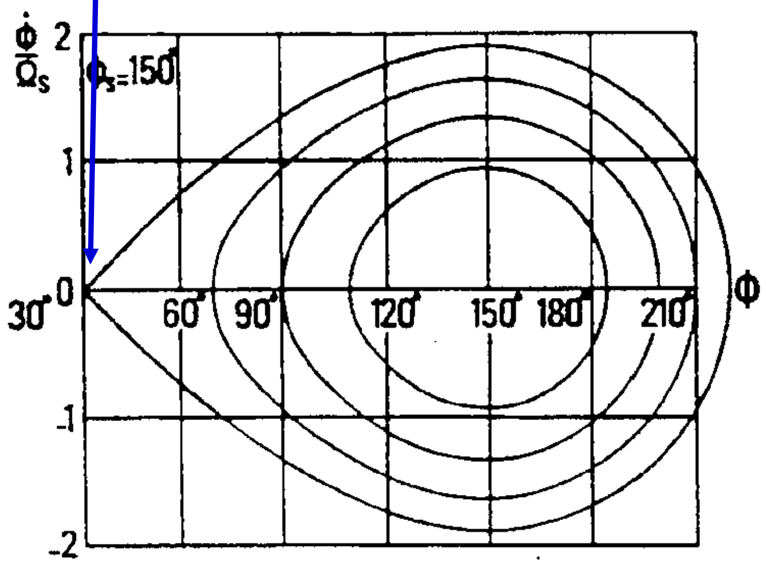
Bucket length I

$$\ddot{\phi} = -\frac{2\pi V_0 h \eta f^2}{E_0 \beta^2 \gamma}\left(\sin\phi - \sin\phi_s\right)$$

The right hand side is negative beyond:

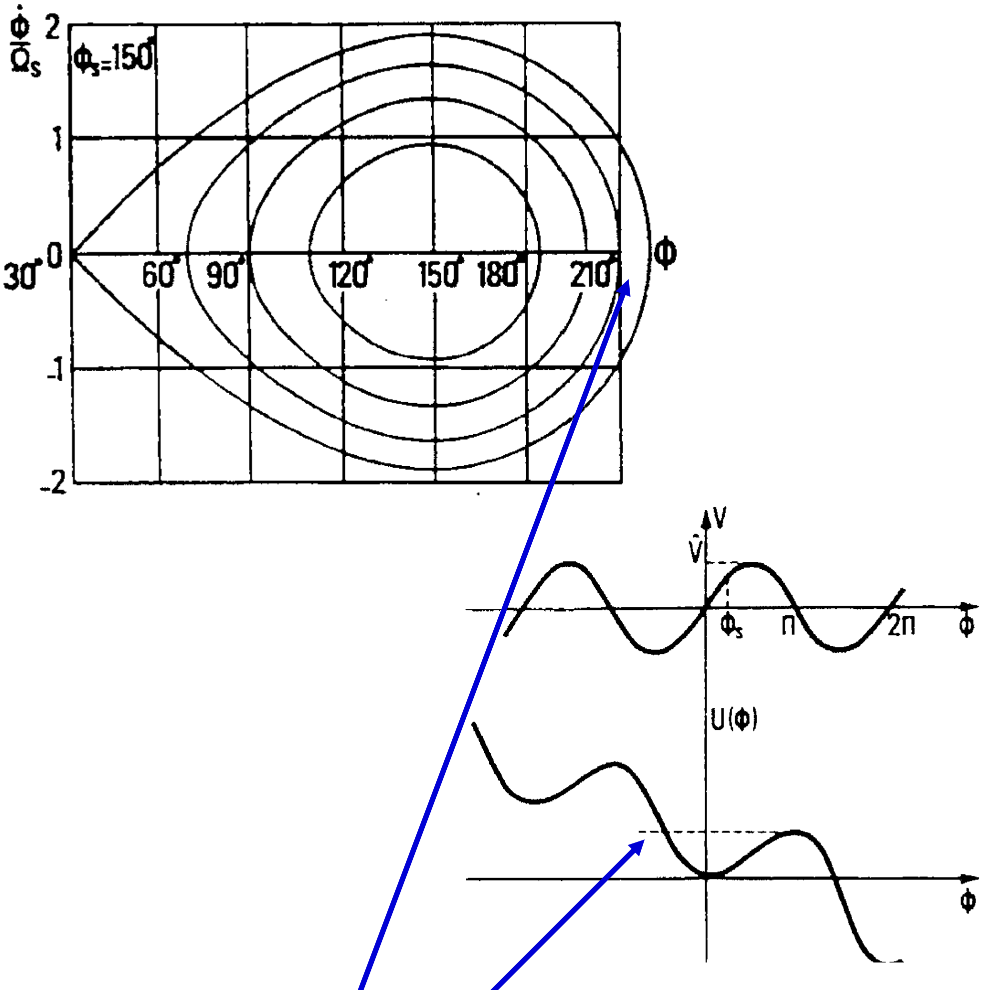
$$\phi = \pi - \phi_s$$





This diagram is flipped left to right

Bucket length II

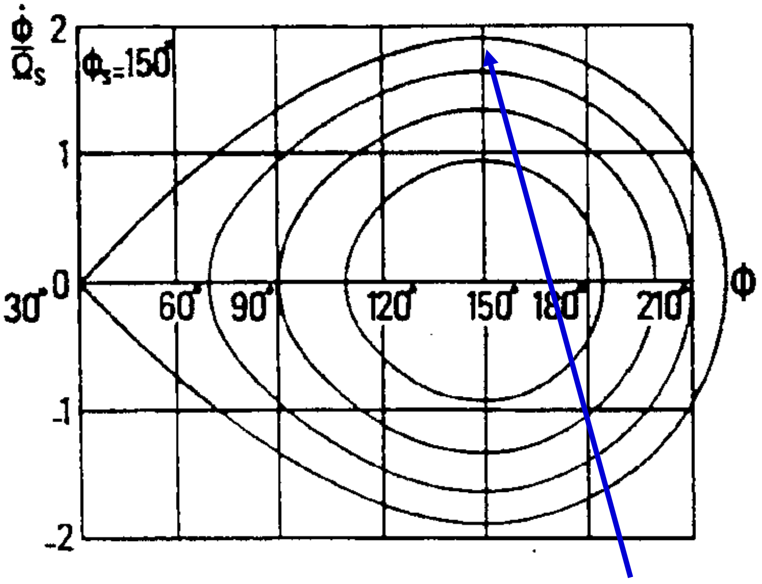


- **And the other limit in phase when at**

$$\phi = \phi_m \quad \text{where}$$

$$\cos\phi_m + \phi_m \sin\phi_s = \cos(\pi - \phi_s) + (\pi - \phi_s)\sin\phi_s$$

Bucket height



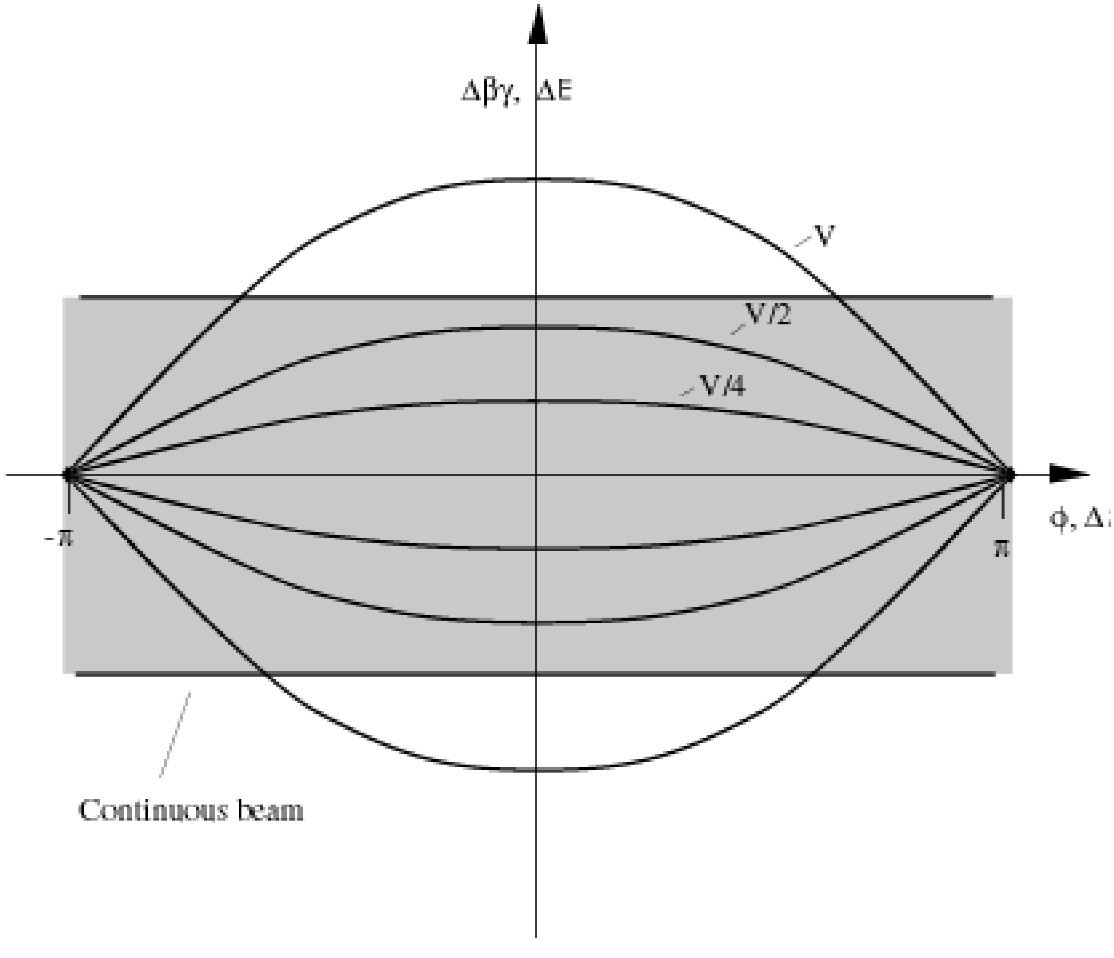
- **And the half height when** $\ddot{\phi} = 0$ **at**

$$\left(\Delta E / E_s \right)_{\max} = \pm \beta \left\{ \frac{eV_0}{\pi h \eta E_s} G(\phi_s) \right\}^{1/2}$$

$$G(\phi_s) = \left[2\cos\phi_s - (\pi - 2\phi_s)\sin\phi_s \right]$$

$$G \quad \text{varies from } \pm 2 \text{ to } 0 \text{ as } \sin\phi_s \text{ varies from } 0 \text{ to } 1$$

Adiabatic capture



- **Area of a stationary bucket is :**

$$A_0 = 16\beta\sqrt{\frac{E_s e V_0}{\pi|\eta|h}} \quad \text{in units} \left[\Delta E.\Delta\phi\right]$$

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